A Study of Inertial Effects in Granular Bed Filtration

The filtration efficiency of a granular bed filter was investigated in the domain where inertial effects are dominant. A filtration model predicting single sphere and total bed efficiencies based on trajectory calculations of dust particles in the voids of the granular bed was developed. The model uses the flow field in a bed of spheres arranged in a body-centered cubic array for which the filtration efficiency was found to be a function of a modified Stokes number defined by $St' = St \times F$ where the function F depends only on the Reynolds number. The value of the function F can be found experimentally or predicted from the Ergun correlation to be $F = 1 + 0.0157\ Re$. The filtration efficiency calculated using the model agrees well with experimental results obtained in beds of dense cubic packing of spheres especially at relatively low Reynolds numbers ($Re < 200$) and relatively high Stokes numbers ($St' > 0.01$). The prediction of the theoretical model also agrees well with experimental data for randomly packed beds at low porosities and low Reynolds numbes.

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SCOPE

Granular bed filters are used for removing micron and submicron size particles from gases. They are especially important in the filtration of hot and corrosive gases where other filters cannot be used. A fundamental understanding of the filtration process in granular beds and the quantitative evaluation of the effects of different parameters on the filtration efficiency are essential in developing and designing an efficient industrial granular filter.

Most theoretical models of filtration in granular beds are based on evaluating the filtration efficiency of a single granule and then integrating the results over the entire bed by assuming that all granules in the bed experience similar filtration conditions and have the same filtration efficiency. The single sphere efficiency is usually determined by calculating the trajectories of aerosol particles flowing toward a single sphere and modifying the gas flow field to account for the porosity of the bed. The major difficulty in using this approach is that the particle trajectories and therefore the filtration efficiency depends strongly on the actual three-dimensional flow profile of the gas flowing through the bed. A representative and accurate three-dimensional velocity profile for flow in a granular bed is not yet availabe in the literature so that simplified flow models must be used for trajectory calculations with results that do not agree well with experimental data.

In the present work an improved version of the Snyder and Stewart (1966) flow model for a dense cubic packing of spheres is used to evaluate the filtration efficiency in a granular bed. The three-dimensional flow model enables trajectory calculations to be made for a single sphere as well as for the entire bed. The calculated single sphere and total bed efficiencies are compared to determine the validity of the single sphere efficiency concept. The results for the single sphere and total bed efficiencies are also compared with experimental data obtained in both dense cubic packing of spheres and in randomly packed beds.

CONCLUSIONS AND SIGNIFICANCE

A filtration model predicting both single sphere and total bed efficiency was developed for a dense cubic packing using the flow field developed by Snyder and Stewart (1966). The model is based on trajectory calculations of dust particles inside the granular bed and assumes no bouncing of dust particles from the surface of the granules. The filtration efficiency was found to be a function of both the Stokes and Reynolds numbers through a modified Stokes number St' defined as $St' = St \times F$ where the factor $F = 1 + 0.0157\ Re$ is derived from the Ergun correlation.

The present work is the first in which a three-dimensional flow field is used for the evaluation of filtration efficiency in granular beds. Trajectory calculations were performed for both a single sphere as well as for many sphere layers in the bed. The efficiencies for the total bed are compared to those for the single sphere corrected for the number of sphere layers or thickness of the bed through the use of Eq. 8 or 9a and show a reasonably good agreement indicating that the single sphere efficiency approach to predict total bed efficiencies is basically valid.

The model predictions agree quite well with experimental results obtained using dense packed beds of spheres at $Re < 200$ and $0.01 < St' < 0.03$. The model predictions also agree well with the experimental correlation by D'Ottavio and Goren (1983) and with our own experimental data for randomly packed beds at low porosities and low Reynolds numbers.

As the kinetic energy of a dust particle increases, either by increasing the velocity or its mass, the deviation of the experimental data from the curve predicted by the model increases and the experimental results are always lower than the theoretical, suggesting that particle bouncing may be occurring. This phenomenon was not considered in the theoretical analysis.

INTRODUCTION

In granular beds high filtration efficiency can be achieved for aerosol particles in the micron and submicron size range. Granular bed filters are especially important in the cleaning of high temperature and corrosive gases, where other filters cannot be used. The parameters affecting filtration efficiency in granular beds are well known, yet the prediction of filtration efficiency is very difficult. There is presently no theoretical model that can accurately predict the filtration efficiency of granular beds from the basic principles of fluid mechanics and mass transfer.

In the granular bed filtration process the dusty gas flows through a bed of granules on which dust particles are captured. The total filtration efficiency of a granular bed is defined as

$$\eta = 1 - n_{out}/n_{in} \tag{1}$$

where n_{out} represents the number concentration of dust particles that penetrate through the entire bed and are found in the filter outlet and n_{in} is the number concentration of particles carried into the bed by the incoming stream. The total efficiency η can be predicted if the trajectories of all dust particles traveling inside the bed are known, and it is assumed that dust particles do not bounce from the granules' surface once they are deposited. If bouncing occurs, a correction must be made for the reentrained particles. Thus the problem of predicting filtration efficiencies, as described above, is basically the problem of predicting the trajectories of dust particles in the bed; a dust particle whose center approaches the surface of a granule to within a distance equal to its radius is said to be collected.

The motion of a dust particle in a granular bed depends on its shape and size, its initial velocity, the flow field in which it travels, the external forces acting on it, and the obstacles in its path. When these parameters are known, the trajectories of the particles traveling inside the bed can, in principle, be computed.

The trajectory equation was presented and used previously by many authors (Paretsky et al., 1971; Spielman and FitzPartick, 1973; Payatakes et al., 1974; George and Poehlein, 1974; Pilat and Perm, 1976; Nielsen and Hill, 1976; Rajagopalan and Tien, 1976; Gutfinger and Tardos, 1979; Fan and Gentry, 1979; Snaddon and Dietz, 1980; Pfeffer et al., 1981; Degani and Tardos, 1981; Pendse and Tien, 1982). For filtration of aerosol particles from gases the equation takes the form

$$\frac{d^2\vec{x}}{dt^2} = \frac{6\pi\mu r_p}{m_p C}\left(\vec{u} - \frac{d\vec{x}}{dt}\right) + \frac{\vec{F}_{ext}}{m_p} \tag{2}$$

where $\vec{x}$ is the position vector, $\vec{u}$ is the gas velocity, r_p and m_p are the particle's radius and mass, C is the Cunningham correction factor, and $d^2\vec{x}/dt^2$ and $d\vec{x}/dt$ are the particle's acceleration and velocity, respectively. $\vec{F}_{ext}$ is a vector summing all external forces acting on a particle traveling inside the bed such as gravitational and electrical forces.

Diffusional and gravitational effects in granular beds are significant only for very small particles ($d_p < 0.5$ μm) at low velocities (Gutfinger and Tardos, 1979). When bed and dust particles are electrically neutral and diffusional and gravitational effects negligible, inertia becomes the dominant mechanism of filtration and Eq. 2 is reduced to

$$\frac{d^2\vec{x}}{dt^2} = \frac{6\pi\mu r_p}{m_p C}\left(\vec{u} - \frac{d\vec{x}}{dt}\right) \tag{3}$$

By nondimensionalizing Eq. 3, it can be easily shown that the filtration efficiency of a granule in the bed is characterized by only one dimensionless group called the Stokes number, $St = 2\rho_p r_p^2 UC/9\mu a$.

Filtration efficiency due to inertial effects can be predicted once the solution of Eq. 3 is known; such a solution is given in the present work. The equation is solved in Cartesian coordinates for a dense cubic packing of spheres (body-centered cubic) at low Reynolds numbers. The theoretical solution is then compared with experimental results obtained in a similar arrangement of spherical particles. The validity of the solution for higher Reynolds numbers and randomly packed beds with different porosities is also examined.

LITERATURE REVIEW

Flow Models in Granular Bed

The actual flow of a fluid in a bed of granules is usually so complicated that simplified models have to be used. These models are usually classified as either internal flows, where the fluid is assumed to flow in pores of different geometry inside the granular bed, or external flows, where the fluid is assumed to flow around the granules (Rajagopalan and Tien, 1979).

In internal flow models the pore is usually taken as a cylindrical capillary or a constricted tube; the pores' walls act as collectors for the dust. Particles reach the surface by various filtration mechanisms. The capillary model was initially used by Jackson and Calvert (1966) for particle collection in a packed bed of spheres. The unit bed element is assumed to have N capillaries per unit area of bed. The main mechanism of filtration in those capillaries, for particles larger than 0.5 micron, is inertial impaction, which occurs due to centrifugal forces acting on the particles as the capillary turns.

More complicated capillary models such as constricted-tube models were developed by Petersen (1958) and more recently by Payatakes et al. (1973) and Niera and Payatakes (1978). The unit cell is characterized by three physical parameters, the maximum and minimum diameter of the constricted tube and its height. The tube wall is assumed to be parabolic (Payatakes et al., 1973), sinusoidal (Fedkiew and Newman, 1977) or hyperboloidal (Petersen, 1958; Venkatesan and Rajagopalan, 1980). Pendse and Tien (1982) found that trajectory calculations using a combination of all three cases mentioned above agree better with experimental data than each of the models alone. The filtration of particles is accomplished by the deposition of particles on the surface of the tube and the efficiency increases as the constriction minimum to maximum diameters ratio decreases. Due to its geometry, the constricted-tube flow model predicts much higher filtration efficiencies than those predicted by the capillary flow model.

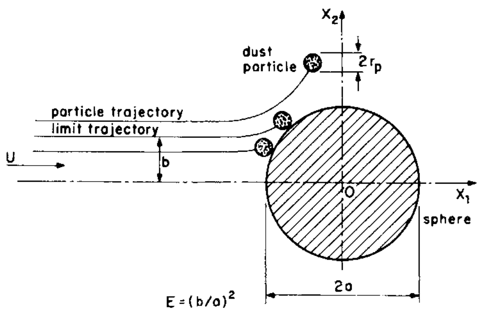

Figure 1. Inertial deposition of particles on a sphere.

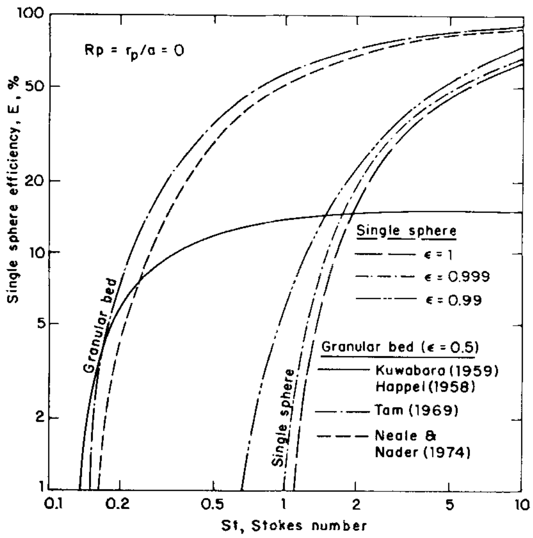

Figure 2. Inertial deposition on a sphere (ϵ = 1) and in a granular bed (ϵ = 0.5). Comparison between different flow models.

Most known external solutions for velocity profiles in granular beds assume the bed to be a homogeneous swarm of spherical granules of uniform size. Each sphere has a free surface (no sphere is touching another), and it is located in a center of a cell surrounded by the flowing fluid. The cells are all identical, experiencing the same flow in their voidage. The radius of the cell, e, is defined so that the void fraction of a unit cell is equal to the local void fraction of the bed

$$e = a(1 - \epsilon)^{-1/3} \tag{4}$$

where a is the radius of the sphere and ϵ is the void fraction in the bed. The flow fields are given in terms of a stream function (Gutfinger and Tardos, 1979) and they differ from each other only by the boundary conditions imposed at the cell boundary. The sphere-in-cell models, such as those suggested by Lamb (1932), Happel (1958), Kuwabara (1959), Tam (1969), and Neal and Nader (1974), assume the flow to be two-dimensional (axisymmetric) and the Reynolds number to be either very small (creeping flow) or very large (potential flow).

Limitations of Existing Flow Models in the Computation of Dust Particle Trajectories

Using two-dimensional flow models for trajectory calculations in a granular bed reduces the vectorial Eq. 3 to a set of two differential equations describing the motion of a dust particle toward a single sphere in the bed. Once the limiting or critical trajectory, i.e., the trajectory that just missed the collector, is found (see Figure 1), the single sphere efficiency can be defined as

$$E = (b/a)^2 \tag{5}$$

where b is the radius of the limiting trajectory. Single sphere efficiencies due to inertia, E, for different flow models, as computed by Tardos (1978), are given in Figure 2. The curves in the figure are based on Eq. 5, where b is calculated from the trajectory equations.

The commonly used flow models (Lamb, 1932; Happel, 1958; Kuwabara, 1959; Tam, 1969; Payatakes et al., 1973; Neal and Nader, 1974) for trajectory calculations in packed beds are far from representative of a real bed. The pressure drop predicted using these models usually agrees reasonably well with experimental data. However, the calculated velocity profiles differ significantly from the actual velocity profiles, since the sphere-in-cell models assume that none of the spheres is touching any other. The existence of contact points between spheres cannot be ignored when calculating dust particles' trajectories. Their existence increases the tortuosity of the stream lines, which strongly affects the motion of a dust particle flowing through the bed. Moreover, many dust particles are captured due to the existence of these contact points, as demonstrated in Figure 3. Dust particles traveling in the plane of the paper can escape through the opening between the spheres if they are nontouching (case A, Figure 3), whereas they would be captured if the two spheres are in contact (case B). Thus filtration efficiency computations based on sphere-in-cell flow models can be expected to predict a lower filtration efficiency than is actually found in a real fixed bed.

The higher filtration efficiency observed experimentally when compared to the theoretical predictions described above is explained by Snaddon and Dietz (1980) to be the result of flow intensification. This intensification is caused by the acceleration of the flow through the constrictions in the voids and is accompanied by flow separation at high Reynolds numbers. The intensified flow impinges onto the upstream face of the bed granules with a velocity greater than the superficial velocity and results in increasing inertial impaction effects. The flow model used by Snaddon and Dietz is the unit-cell potential flow model (Lamb, 1932), where the original boundary conditions on the cell surface are replaced by

$$U_r(r = 1)\begin{cases} = -\beta U \cos\theta & 0 \leq \theta \leq \phi \\ =0 & \phi < \theta \leq \pi/2 \end{cases} \tag{6}$$

where

$$\phi = 1/2 \cos^{-1}(1 - \beta/2) \tag{7}$$

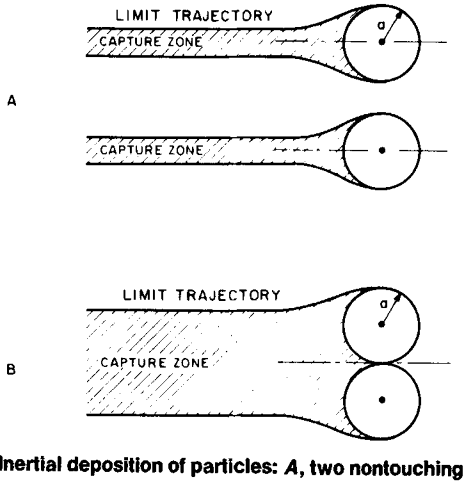

Figure 3. Inertial deposition of particles: A, two nontouching spheres; B, two touching spheres.

TABLE 1. EMPIRICAL CORRELATIONS FOR SINGLE SPHERE EFFICIENCY

Author	E	Range
Paretsky (1972)	$2 \times St^{1.13}$	$St < 0.01$
Meisen and Mathur (1974)	$0.00075 + 2.6 \times St$	$St < 0.01$
Doganoglu (1975)	$2.89 \times St$	$d_g = 100$ micron
	$0.0583 \times Re \times St$	$d_g = 600$ micron
Thambimuthu et al. (1978)	$10^5 \times St^3$	$0.001 < St < 0.01$
Schmidt et al. (1978)	$3.75 \times St$	$St < 0.05$
Goren (1978)	$1270 \times St^{9/4}$	$0.001 < St < 0.02$
Pendse and Tien (1982)*	$(1 + 0.04\,Re)[St + f(R_p)]$	
D'Ottavio and Goren (1983)†	$\dfrac{St_{eff}^{3.55}}{1.67 + St_{eff}^{3.55}}$	$13 < Re < 1650$; $0.33 < \epsilon < 0.38$

* $f(R_p) = 0.48\left(4 - \frac{4}{d_c}R_p - R_p^2/d_c^2\right)^{1/2}(R_p^{1.041}/d_c)$

† $St_{eff} = f(Re,\epsilon)St$

$f(Re,\epsilon) = (1 - h^{5/3})/(1 - 1.5h^{1/3} + 1.5h^{5/3} - h^2) + 1.14Re^{1/2}/\epsilon^{2/3}$

where $h = 1 - \epsilon$

and where β, the intensification factor, is unity for very low Reynolds numbers and increases with increasing Reynolds number. As β increases, ϕ decreases, and the flow is constrained to enter the cell in a narrowing jet centered around the axis. This model, however, is semiempirical since β must be evaluated experimentally.

Lately, a number of researchers have developed empirical correlations for the single sphere efficiency, especially for the region where inertial effects are dominant; some of these correlations are given in Table 1. These correlations and theoretical predictions of filtration efficiency based on the sphere-in-cell model for creeping and potential flow are compared in Figure 4. The large discrepancy between the results indicates that the flow models described above are not sufficiently representative for use in trajectory calculations. A similar conclusion was reached regarding the constricted-tube models from the comparison of experimental and theoretical data by Pendse and Tien (1982).

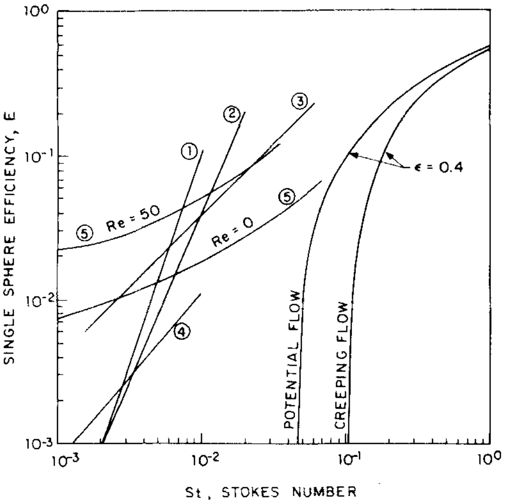

Figure 4. Theoretical predictions of single sphere efficiency in a granular bed (Tardos and Pfeffer, 1980) compared with empirical correlations:

1, Thambimuthu et al. (1978), $\epsilon = 0.36$, $Re = 2\text{-}20$

2, Goren (1978), $\epsilon = 0.4$, $Re = 20\text{-}125$

3, Schmidt et al. (1978), $\epsilon = 0.44$, $Re = 30\text{-}300$

4, Paretsky et al. (1971), $\epsilon = 0.41$, $Re = 30\text{-}250$

5, Pendse and Tien (1982), $\epsilon = 0.38$, $R_p = 0.003$

Total Filter Bed Efficiency

Assuming that all filter elements in a granular bed experience similar filtration phenomena, the total bed efficiency, η, can be found by summing the effect of all elements in the bed. The semiempirical expression commonly used for the efficiency η is

$$\eta = 1 - \exp[-K_1(1 - \epsilon)(L/2a)E] \tag{8}$$

where E is the single sphere efficiency, defined as the ratio of the number of all aerosol particles captured by a single sphere in the bed to the total number of particles flowing toward it in a circular tube of cross-sectional area πa^2, where a is the radius of the sphere (see Figure 1), ϵ is the bed porosity, and K_1 is a constant. The values given for K_1 are 1.5 (Snaddon and Dietz, 1980; D'Ottavio and Goren, 1983); $1.5/\epsilon$ (Paretsky et al., 1971; Tardos et al., 1974); $1.5f$, where f is a collision factor (Patterson and Jackson, 1977); and 1.875, an empirical constant (Schmidt et al., 1978).

Another expression used by Pendse and Tien (1982) to obtain the total bed efficiency of a granular bed composed of n' identical unit cells in series is

$$\eta = 1 - (1 - E')^{n'} \tag{9a}$$

where E' is the efficiency of a unit cell, defined as the ratio of the number of aerosol particles captured by the sphere to the total number of particles flowing toward it in a square duct of cross-sectional area ℓ^2, where ℓ is a characteristic length of the unit cell. For a granular bed of spherical particles this quantity is given by

$$\ell = \left(\frac{\pi}{6(1 - \epsilon)}\right)^{1/3} d_g \tag{9b}$$

and is seen to be a function of voidage and grain diameter. For a randomly packed bed of spheres of voidage $\epsilon = 0.4$, $\ell = 0.956d_g$; for a dense cubic array ($\epsilon = 0.26$), $\ell = 0.891d_g$. The number of unit cells or layers, n', in Eq. 9a is related to ℓ by the relation

$$n' = L/\ell \tag{9c}$$

where L is the total height of the bed.

It was shown by Clift et al. (1981) and also recently by Tien (1984) that for small values of E, the ratio E'/E is not always unity but is given by the expression ($K_1 = 1.5$ in Eq. 8)

$$E'/E = \pi^{1/3}[0.75(1 - \epsilon)]^{2/3} \tag{9d}$$

For a randomly packed bed of collecting spheres of $\epsilon = 0.4$, $E'/E = 0.86$; for a dense cubic array ($\epsilon = 0.26$), $E'/E = 0.99$. Thus the difference between E' and E can be numerically significant depending on the bed voidage, but there is essentially no difference between them for the dense cubic array that is used as the flow model in this paper. Since there is no single accurate and universally accepted solution for either E or E', there is no way to confirm the validity of either Eq. 8 or 9a. Experiments do show (Knetting and Beeckmans, 1974; Doganoglu et al., 1978; D'Ottavio and Goren, 1983) that for large values of $L/2a$ there exists an exponential dependence between $L/2a$ and the efficiency η. The effect of the other parameters on η is not quantitatively clear or agreed upon. In spite of the limitations of the sphere-in-cell models and the single sphere efficiency approach they are widely used in the literature

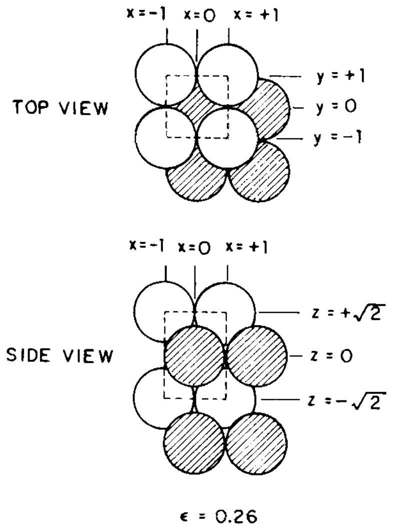

Figure 5. Schematic representation of a dense cubic packing of spheres.

(Paretsky et al., 1971; George and Poehlein, 1974; Tardos et al., 1974; Pilat and Perm, 1976; Gutfinger and Tardos, 1979; Nielsen and Hill, 1976; Dietz, 1981; Lee and Gieseke, 1979; Tardos and Pfeffer, 1980; Pfeffer et al., 1981; Degani and Tardos, 1981; Pendse and Tien, 1982; D'Ottavio and Goren, 1983) mainly because no other solutions for the filtration efficiency of granular beds are available.

AEROSOL FILTRATION IN A DENSE CUBIC BED OF GRANULES

Flow Model

In order to predict the trajectory of dust particles in packed beds, a more realistic flow model is needed. Flow models for arbitrarily arranged spheres in packed beds are not yet available in the literature. There are, however, three-dimensional models of flow through periodic arrays of spheres in special arrangements such as dense cubic, simple cubic, and face-centered cubic packings. These models, which are valid for low Reynolds numbers, were developed by Snyder and Stewart (1966), Sorenson and Stewart (1974), and more recently by Sangani and Acrivos (1982) and Zick and Homsy (1982). In this paper Snyder's flow model for flow in a dense cubic bed is used for trajectory calculations. The model has the advantage of describing the flow in a bed that can be readily constructed; thus theoretical predictions of filtration efficiency based on trajectory calculations can be readily compared to experimental data.

The fluid velocity profile and the pressure distribution at low Reynolds number for the dense cubic packing (Figure 5) was obtained by Snyder and Stewart (1966) and is given by

$$v_j = \sum_{i=1}^{N} C_{ji}\chi\phi_{ji} \quad j = 1,2,3 \tag{10}$$

$$P = (Z + \sqrt{2})/2\sqrt{2} + \sum_{i=1}^{N} C_{pi}\phi_{pi} \tag{11}$$

where j are the three Cartesian coordinates, C_{ji} and C_{pi} are constants, χ is a function that satisfies the no-slip condition on M neighboring spherical surfaces, and ϕ_{ji} and ϕ_{pi} are combinations of trigonometric functions satisfying the periodicity and symmetry conditions. Here v_j is a dimensionless fluid velocity defined by

$$v_j = u_j\mu/a\Delta p \tag{12}$$

where u_j is the velocity in the j direction, μ is the viscosity of the fluid, a is the radius of a sphere, and Δp is the pressure drop over a unit cell that extends from $Z = -\sqrt{2}$ to $Z = \sqrt{2}$. The dimensionless superficial velocity in the z direction is similarly defined as $V = (\mu/a)(U/\Delta p)$, where U is the superficial velocity in the z direction. The value of V extrapolated to zero Reynolds number, $Re \rightarrow 0$

$$V_{Re\rightarrow 0} = V_0 = (\mu/a)(U/\Delta p)_{Re\rightarrow 0} = 0.000258$$

can be found from the experimental work by Martin et al. (1951).

The constants C_{ji} and C_{pi} in Eqs. 10 and 11 can be evaluated by using Galerkin's method. The trial solutions are substituted into the continuity and momentum equations for $Re = 0$ and integrated over the volume τ of a unit cell

$$\int_\tau \sum_{j=1}^{3} \frac{\partial v_j}{\partial x_j} w_i' \, d\tau = 0 \quad i = 1,2,\dots N \tag{13}$$

$$\int_\tau \left(\nabla^2 v_j + \frac{\partial P}{\partial x_j}\right) w_i \, d\tau = 0 \quad i = 1,2,\dots N \quad j = 1,2,3 \tag{14}$$

Here w_i and w_i' are weight functions defined in Snyder and Stewart (1966) and the integration is done numerically using M-triple Gaussian points. Introducing the trial functions, given in Eqs. 10 and 11, into Eqs. 13 and 14 and integrating over the volume of the cell reduce the system into $4 \times N$ linear equations with C_{ji} and C_{pi} as unknowns. The velocity profile satisfies the conservation equations approximately and the boundary and symmetry conditions exactly. The solution is given in detail by Snyder (1965).

The dense cubic packing of spheres is shown in Figure 5. The unit cell enclosed by dotted lines contains the sphere centered at (0,0,0) and one-eighth portion of eight adjacent spheres. The void fraction of this arrangement is 0.26 and the flow is assumed to be in the positive z direction. A necessary condition for the solution to be accurate is that $V^*(Z)$, which is the average normalized dimensionless velocity (normalized by $V_0 = 0.000258$) in the z direction in the range $-\sqrt{2} < Z < \sqrt{2}$ given by

$$V^*(Z) = 1/4 \int_{-1}^{1}\int_{-1}^{1} (v_z/V_0)\, dX\, dY \tag{15}$$

should be unity. The dimensionless velocity $V^*(Z)$, as obtained using Snyder's 22-term solution in the range $0 < Z < \sqrt{2}$, is shown in Figure 6; the solution deviates by +11.8% to −21.4% from the required value of unity. This error is clearly too large for trajectory calculations. Improvement in accuracy was achieved in the present

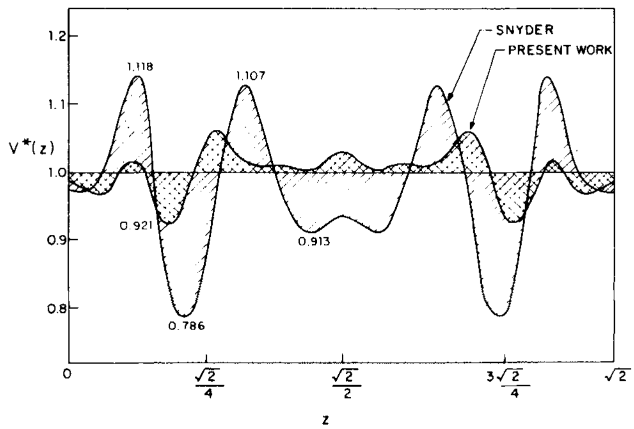

Figure 6. Velocity $V^*(Z)$ as defined by Eq. 15. Comparison between Snyder's solution and the present work.

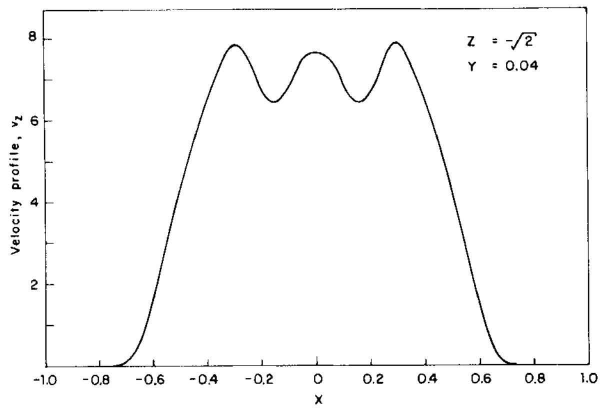

Figure 7. Velocity v_z as a function of X at $Z = -\sqrt{2}$, $Y = 0.04$ in the range $-1.0 < X < 1.0$.

work by increasing the number of terms in the series from 22 terms to 68. Using this solution with 20-triple Gaussian points (compared to the 16 used by Snyder), $V^*(Z)$ converges to 1.0 with a deviation of only 5.9% to −7.9%, as shown in Figure 6. This solution for the velocities v_x, v_y, v_z and the pressure P requires the solution of a 272 × 272 matrix to determine the coefficients C_{xi}, C_{yi}, C_{zi}, and C_{pi}. The convergence of the series is very slow, and in order to increase the accuracy even further one has to solve for many more constants, a procedure that requires an enormously large computer capacity. For the trajectory calculations presented in this work, however, the 68-term solution seems to be reasonably accurate. A typical velocity profile v_z in the plane $Z = -\sqrt{2}$ as obtained by using the 68-term solution is given in Figure 7.

Trajectories in a Dense Packed Bed

The flow model described above can be used in the trajectory equation (Eq. 3) to predict filtration efficiencies in a dense cubic packing of spheres. In dimensionless form Eq. 3 has the form

$$\frac{d^2\vec{X}}{dT^2} = \frac{1}{StF}\left(\vec{U} - \frac{d\vec{X}}{dT}\right) \tag{16}$$

where the dimensionless parameters are

$$\vec{X} = \vec{x}/a$$
$$T = tUF/a$$
$$\vec{U} = \vec{u}/UF$$
$$St = 2\rho_p r_p^2 UC/9\mu a.$$

The factor F is defined as the ratio of the dimensionless superficial velocity at $Re \rightarrow 0$, V_0, to the superficial velocity at any Reynolds number

$$F = V_0/V = (U/\Delta p)_{Re\rightarrow 0}/(U/\Delta p)_{Re\neq 0} \tag{17}$$

The velocities V and V_0 can be found experimentally or by using the experimental data by Martin et al. (1951). F can also be determined for any granular arrangement using the Ergun correlation

$$\frac{\Delta P}{L} = \frac{150\mu U(1-\epsilon)^2}{d_g^2\epsilon^3} + \frac{1.75\rho U^2}{d_g}\frac{(1-\epsilon)}{\epsilon^3} \tag{18}$$

where ΔP is the total pressure drop across the bed, $L = d_g(km + 1)$ is the thickness of the bed, m is the number of unit cells in the z direction (see Figure 10), and k is a factor correlating the height of a unit cell to the diameter of the spheres, e.g., $k = \sqrt{2}$ in a dense packed bed, $k = 1$ for a simple cubic arrangement, and so on. Using Eq. 18 one can find the value of V to be

$$V = \frac{U\mu}{\Delta pa} = \frac{2\epsilon^3}{[150(1-\epsilon) + 1.75Re](k + 1/m)(1-\epsilon)} \tag{19}$$

and

$$F = V_0/V = 1.0 + 1.75Re/150(1-\epsilon) \tag{20}$$

Using the Ergun correlation, the velocity V_0 for the dense cubic packing (with $k = \sqrt{2}$, $\epsilon = 0.26$, $Re \rightarrow 0$, $m \rightarrow$ large) is 0.0003, which is about 16% higher than the experimental value of Martin et al. (1951). The value of F is unity for $Re = 0$ but becomes larger for higher values of the Reynolds number. The product $St \times F$ in Eq. 16 is defined as a modified Stokes number St' and is given by

$$St' = St \times F = St[1.0 + 1.75Re/150(1-\epsilon)] \tag{21}$$

Introducing the modified Stokes number St' into Eq. 16 yields

$$\frac{d^2\vec{X}}{dT^2} = \frac{1}{St'}\left(\vec{U} - \frac{d\vec{X}}{dT}\right) \tag{22}$$

Equation 22 is used for trajectory computations with the initial conditions:

$$\vec{X}\ (T = 0) = (X_0, Y_0, -\sqrt{2}) \tag{22a}$$

$$d\vec{X}/dT\ (T = 0) = \vec{U} \tag{22b}$$

which assumes that the velocity of the particle is identical to the velocity of the gas at $T = 0$. The actual initial velocity of the dust particles at $Z = -\sqrt{2}$ depends on the Stokes number: It is exactly equal to the gas velocity $\vec{U}$ for $St = 0$ but is different for larger Stokes numbers. Thus, using the initial conditions as shown above introduces a small error in the trajectory calculation for small Stokes numbers that increases, however, with increasing Stokes number.

As a first step single sphere efficiencies in the unit cell were computed using Eq. 22. The flow in the cell is taken to be in the positive z direction and all dust particles are assumed to be entering the first layer of cells through the plane at $Z = -\sqrt{2}$. The single sphere collection efficiency, E, is defined as the ratio of the flux of dust particles through the area from which they are collected, A_c (Figure 8) to the flux of particles flowing toward the cell through the area A_o equal to the cross-sectional area of the cell

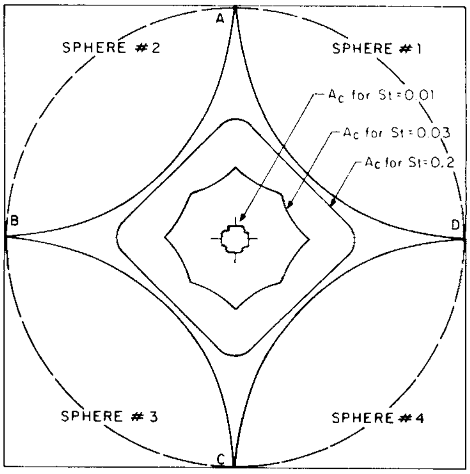

Figure 8. Area A_c at $Z = -\sqrt{2}$ from which dust particles are collected on the central sphere of the cell for different Stokes numbers.

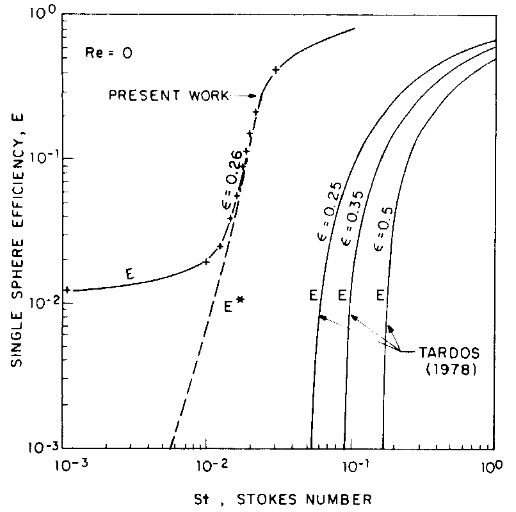

Figure 9. Comparison of single sphere efficiencies due to inertia as obtained in the present work and by Tardos (1978).

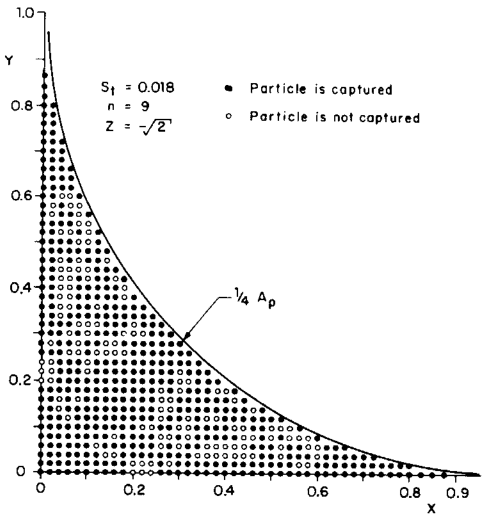

Figure 11. Computational grid in the area A_p with $dX = dY = 0.02$ for a nine-layer bed at $St = 0.018$.

$$E' = \frac{1}{c_0 U_{po} A_0} \int_{A_c} c\vec{u}_p dA \tag{23}$$

Here c and u_p are the dust concentration and the dust velocity, respectively, at $Z = -\sqrt{2}$, and c_o and U_{po} are the uniform dust concentration and velocity far outside the bed. The filtration efficiency of the first half-layer of spheres, the half facing the stream, is assumed to be insignificant so that all dust particles flowing toward the cell are present at the plane $Z = -\sqrt{2}$ where the trajectory computation starts. The error involved in this assumption was demonstrated by experiment to be small. The assumption also stands to reason based on the fact that these half-spheres can be considered as single spheres in an almost infinite fluid (ϵ is very large) where the collection efficiency for Stokes numbers $St < 0.1$ is very small.

Assuming that no particles are captured at the entrance of the cell, that the particle velocity is equal to the gas velocity $\vec{u}_p = \vec{u}$, $u_{po} = U$, the coefficient $F = 1$ ($Re \rightarrow 0$), and that the concentration $c = c_o$ at $Z = -\sqrt{2}$, Eq. 23 can be rewritten as

$$E = \int_{A_c} \vec{U} dA \Big/ \int_{A_p} \vec{U} dA = \frac{1}{A_0} \int_{A_c} \vec{u} dA \tag{24}$$

where A_p is the open area of the cell at $Z = -\sqrt{2}$ (see Figure 8). The area A_c for which the integration is performed is found from the solution of the limiting trajectories. The size of the area A_c depends on the Stokes number and the initial conditions of the dust particles entering the bed. Computation of a trajectory is stopped when the center of a dust particle approaches to within a dimensionless distance R_p of the surface of a sphere where it is said to be

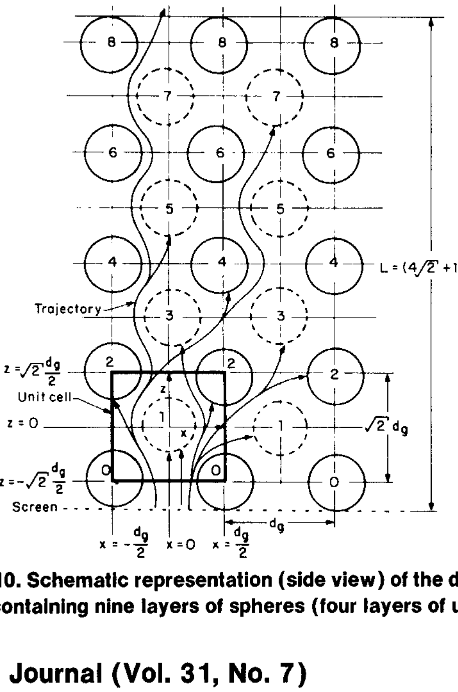

Figure 10. Schematic representation (side view) of the dense cubic bed containing nine layers of spheres (four layers of unit cells).

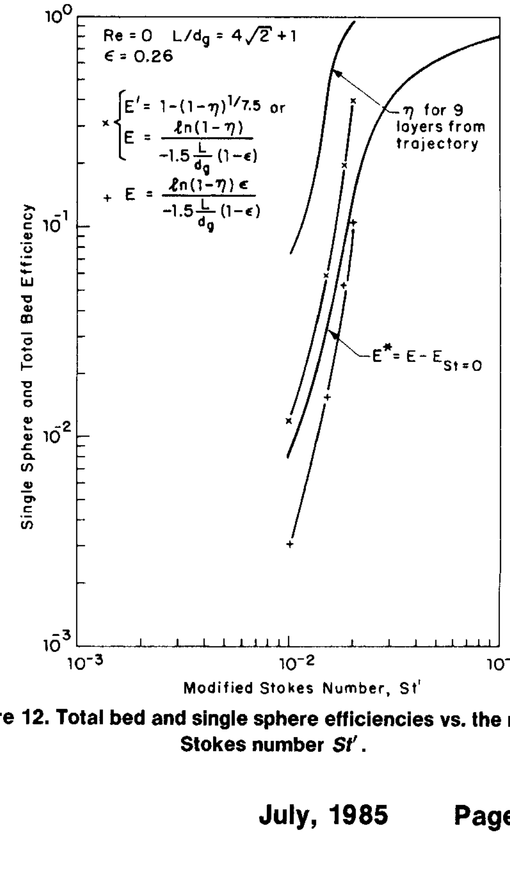

Figure 12. Total bed and single sphere efficiencies vs. the modified Stokes number St'.

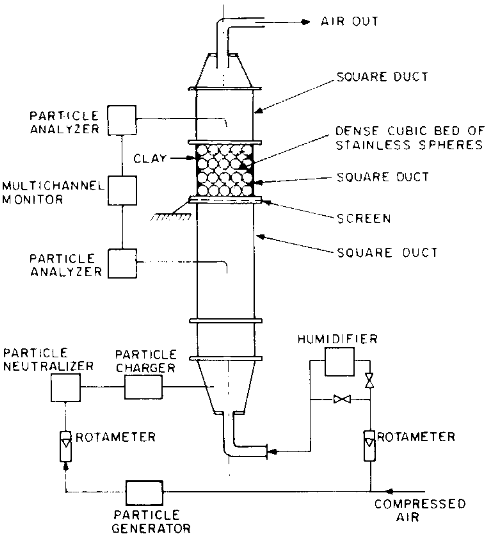

Figure 13. Schematic representation of the experimental setup.

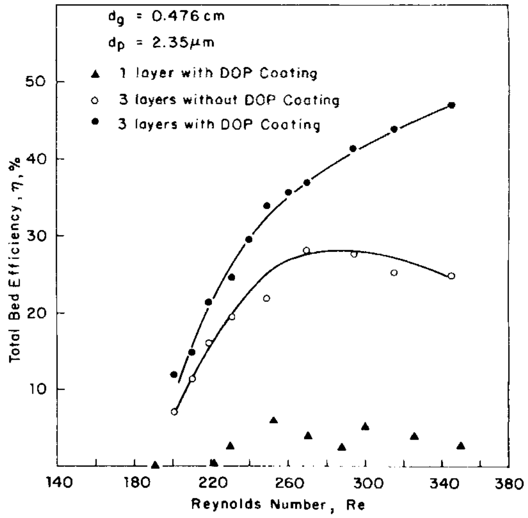

Figure 14. Total bed efficiency as a function of Reynolds number for a dense cubic bed of spheres with and without DOP coating.

collected or when a particle leaves the cell. In such computations, interception, determined by the value of the parameter R_p, is usually small compared to inertial effects unless the Stokes number, St, is very small. The area A_c for the central sphere in the cell for Stokes numbers, $St = 0.2$, 0.03, and 0.01, is given in Figure 8 and the efficiency E for different Stokes numbers is given in Figure 9. In the same figure single sphere efficiencies computed by Tardos (1978) using the Neal and Nader (1974) flow model (for $\epsilon = 0.25$, 0.35, 0.5) are also given. As expected the efficiency found in the present work is much higher than that predicted by the sphere-in-cell models. The two main reasons for the higher efficiency are

1. The effect of contact points between spheres is taken into account and the contact points (12 for each sphere in the cubic dense bed) increase the tortuosity of the flow and hence the inertial effects.
2. Flow intensification effects that arise in the present model because of the assumed structure of the bed. The flow enters the cell through the cross-sectional area A_p ($ABCD$) shown in Figure 8. The main flux is in the center of this area where the average velocity is much higher than the superficial velocity in the bed (see Figure 7). The flux is thus targeted toward the central sphere of the cell and impinges on its upstream face.

The velocity profile given by Eq. 10 satisfies the boundary and symmetry conditions exactly and the conservation equations approximately. The error in the flow field introduces an error in the trajectory computations. A simple way to estimate the error was to compute the filtration efficiency at zero Stokes number, $St = 0$, for particles of dimensionless radius, $R_p = 10^{-5}$. The actual efficiency for these conditions should be very small since at $St = 0$ particles travel exactly along stream lines and at $R_p = 10^{-5}$ the interception effect is negligible. The calculated efficiency, however, is 1.1% for the sphere situated in the center of the cell as well as for an entire bed composed of nine layers of spheres. This result indicates that some stream lines originating from around the point $(0,0,-\sqrt{2})$ are on a collision path with the sphere centered at

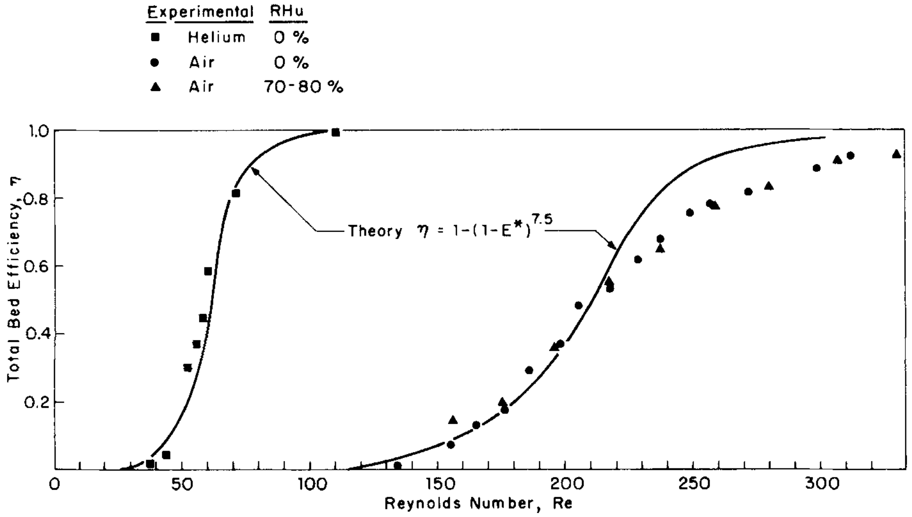

Figure 15. Total bed efficiency as a function of Reynolds number for a nine-layer dense cubic bed of spheres. The curves represent trajectory solutions.

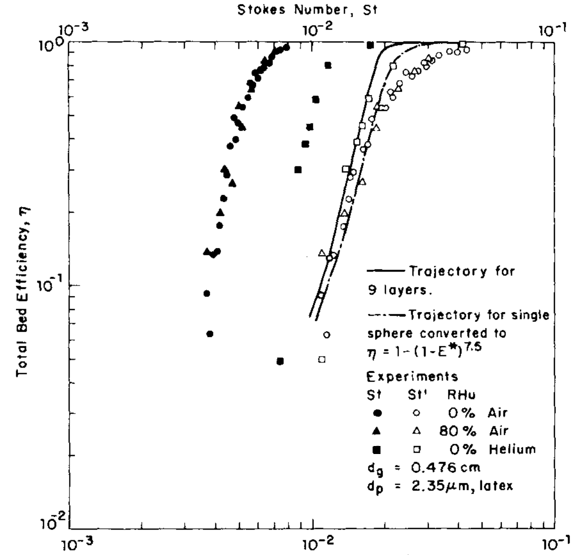

Figure 16. Total bed efficiency as a function of Stokes number and modified Stokes number for a nine-layer dense cubic bed of spheres. The curves represent trajectory solutions.

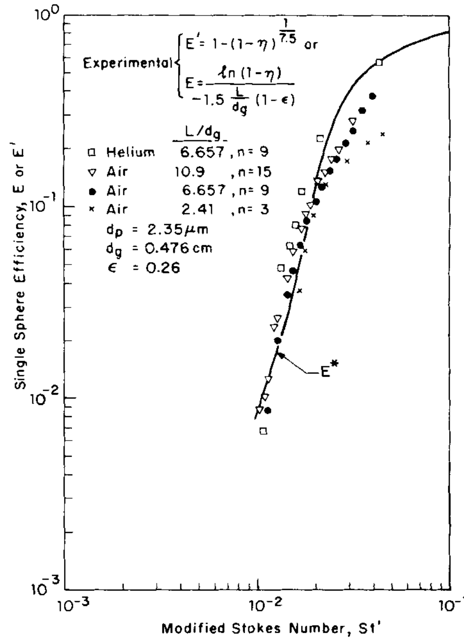

Figure 17. Single sphere efficiency as a function of modified Stokes number; theoretical prediction and experimental data.

(0,0,0). This error is small for large Stokes numbers where the efficiency is high compared to 1.1%, but it becomes large and even dominant for very low Stokes numbers where the filtration efficiency is very small. To overcome this problem we define the efficiency E^* as

$$E^* = E - E_{St=0} \tag{25}$$

to insure zero efficiency at zero Stokes number. The efficiency E^* is shown as the dashed line in Figure 9. A correlation $E^* = f(St')$ was found by linear regression of the computed values in Figure 9 and is given by

$$E^* = \frac{2St'^{3.9}}{4.3 \cdot 10^{-6} + St'^{3.9}} \tag{26}$$

for $0.1 < St' < 0.03$. The correlation fits the computed data within 7% and it gives similar values to the experimental correlation of D'Ottavio and Goren (1983) (see Table 1) for $0.01 < St < 0.02$ and low values of the Reynolds number.

The total bed efficiency, η, can be computed using a similar procedure as described above for any number of unit cells situated on top of each other. A schematic of a bed made of nine layers (4 unit cells) is shown in Figure 10. The trajectory starts at $Z = -\sqrt{2}$ and the origin of the system is at (0,0,0), which is the center of the central sphere in the first cell. A particle not collected by the first cell (layers 0–2) leaves it at $Z = \sqrt{2}$. For computation purposes the origin of the coordinate system is then transferred to $(0,0,2\sqrt{2})$ and the trajectory computation continues. From the particle's point of view it is entering the second unit cell. For the computer program the particle is back at the plane $Z = -\sqrt{2}$. The initial conditions at the inlet to the second cell are the outlet conditions from the first cell. Hence, for a large number of layers, the error introduced by the assumed initial conditions in Eq. 22 at $Z = -\sqrt{2}$ becomes small. If the particle leaves the cell at $X = 1$ the origin is then transferred to (2,0,0) and the computation is continued as mentioned above. The computation is stopped when the dust particle is collected on one of the spheres or when it passes through a predetermined number of layers.

The area A_c used in Eq. 24 to obtain the filtration efficiency of an entire bed (E is changed to η) is composed of many subregions in the initial $Z = -\sqrt{2}$ plane corresponding to the many spheres on which the dust particles are collected, and this area is shown for $St = 0.018$ in Figure 11. The nodal points of the grid shown in Figure 11 are used as starting points for trajectory computations. From symmetry considerations only 1/8 of the total area A_p has to be searched for particles that are not collected. The error, for the computation, depends on the distance between nodal points. The total efficiency, η, for the bed shown in Figure 10 for Stokes numbers St = 0.01, 0.015, 0.018, and 0.02 using node increments of $dX = dY = 0.01$ is shown in Figure 12. The numerical error in evaluating the integral in Eq. 23 is estimated at less than 5% of the total efficiency based on reducing the increments dX and dY from $dX = dY = 0.02$ to $dX = dY = 0.01$. In the same figure, the single sphere efficiency E, calculated from the values of the total bed efficiency, η, for nine layers using Eq. 8 with $K_1 = 1.5$, and the single sphere efficiency E', using Eq. 9a with $n' \approx 7.5$ (Eq. 9c) are compared to single sphere efficiencies obtained directly from trajectory computations for the central sphere in a unit cell. Since E'/E for the dense cubic array is 0.99 (see Eq. 9d) both are represented by a single curve. Equation 8 with $K_1 = 1.5/\epsilon$ is also plotted and shows reasonably good agreement with E^* for $St = 0.018$–0.02; for lower Stokes numbers, however, the single sphere efficiency is much too low. It is not clear from these results whether Eq. 8 or 9a is the more appropriate for total bed efficiency calculations, and therefore this question will be answered by a comparison with experimental data.

EXPERIMENTAL APPARATUS AND PROCEDURE

To examine the validity of the filtration model, experiments were conducted in which a densely packed cubic bed was used to filter solid latex particles from both an air and a helium stream. A schematic drawing of the apparatus used in these experiments is shown in Figure 13. The bed was constructed of uniform metallic spheres contained in a rectangular duct where the spheres could be arranged exactly as described by the model (see Figure 10). The granule diameter d_g was 0.476 cm and the cross-sectional area of the duct $5.23 \times 5.23\ cm^2$ to allow for exactly 11×11 spheres in all odd layers. There were 10×10 spheres in even layers and the excess voidage near the wall was blocked by clay to avoid flow along the walls. The actual cross-sectional area of the bed was therefore $4.76 \times 4.76\ cm^2$.

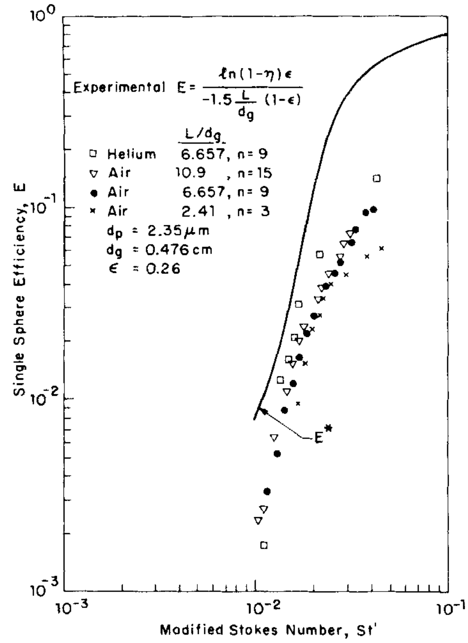

Figure 18. Single sphere efficiency as a function of the modified Stokes number; theoretical prediction and experimental data.

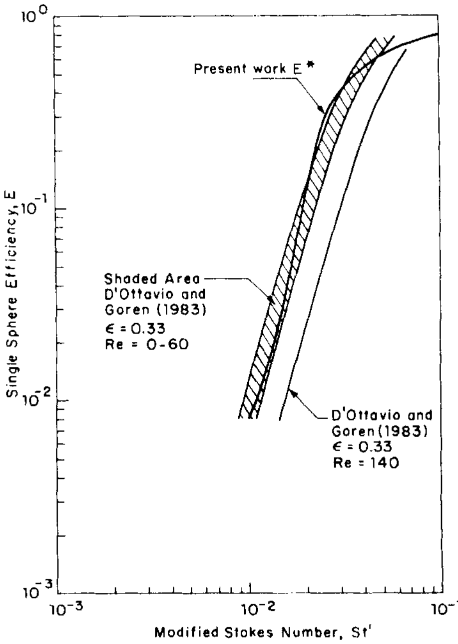

Figure 19. Single sphere efficiency as predicted for a dense cubic packing compared to an empirical correlation for a randomly packed bed ϵ = 0.33, *Re* = 0–60, 140; from D'Ottavio and Goren (1983).

Experiments with a bed thickness equivalent to n = 3, 9, and 15 layers of spheres (m = 1, 4, and 7 unit cells) were conducted. The bed was grounded to avoid buildup of electrostatic charge on the spheres. The granules' surface was coated in most experiments with a DOP layer to reduce bouncing.

Monodispersed latex particles were generated from a solution of pure methanol and their concentration was measured simultaneously at the inlet and outlet of the bed using Climet CI-225 and CI-208S as sensors and a Climet CI-210 multichannel monitor. The filtration efficiency was also measured using a Royco-1200 sensor with a Royco-4100 monitor. The difference between the readings on the Climet and Royco monitors was always less than 5%. The gas humidity in the system was controlled and measured and experiments at relative humidities varying from 0 to 80% were performed.

The total length of the square duct was 58 cm below the bed and 30 cm above the bed to allow for fully developed flow in the duct and a good mixing of the aerosol in the gas. Experiments with gas flowing upward (as shown in Figure 13) and with gas flowing downward were performed with no apparent difference in filtration efficiency, indicating that capture by gravity settling was negligible. The gas velocity was controlled using rotameters and ranged between 40 and 100 cm/s for air (Re = 120–330) and between 90 and 270 cm/s for helium (Re = 35–110).

Filtration experiments using randomly packed beds of glass spheres were also performed to examine the validity of our results for random packings. The experimental conditions were: diameter of the glass spheres d_g = 0.125 cm, ϵ = 0.37, air velocities 25–125 cm/s (Re = 20–100), and a bed thickness of 3.17 cm.

EXPERIMENTAL RESULTS AND DISCUSSION

The total bed efficiency as defined in Eq. 1 was found experimentally and is plotted against Reynolds number in Figures 14 and 15 and against Stokes number in Figure 16. No apparent differences in filtration efficiency were observed for different relative humidities in the range between 0 and 80% at room temperature. These results indicate that no electrostatic effects were involved and that inertia was the dominant mechanism of filtration during the experiments.

In most experiments the spheres were coated with DOP to reduce bouncing. The effect of the coating is shown in Figure 14. The efficiency of the clean bed (spheres were washed with ethanol) is much lower than that of a bed sprayed with DOP. While the efficiency of the latter bed increases continuously with velocity, the clean bed efficiency reaches a maximum and then drops as the velocity increases. Although coating the granules' surface with DOP reduces bouncing significantly, it is not clear if bouncing is eliminated completely.

The efficiency of the screen and the first layer of spheres is assumed negligibly small. This assumption was examined by an experiment where only the screen and one layer of spheres were used to filter solid latex particles. The results shown in Figure 14 indicate that the assumption is indeed correct; the maximum efficiency of the screen and the one layer of spheres was 6% with most of the results around 4% and lower, which is well within the error of the measuring device.

Air and helium were used in the experiments. Helium has a viscosity similar to that of air but a density of only about one-seventh that of air. Thus, for the same Stokes number, the Reynolds numbers for the experiments with helium are much smaller than that of air. As shown in Figure 15 high filtration efficiency is obtained using helium at much lower Reynolds numbers as compared to air. On the other hand, as shown in Figure 16, higher efficiency was obtained using air at much lower Stokes numbers as compared to helium. The filtration efficiency therefore must clearly be a function of both the Stokes and the Reynolds numbers. The parameter St', which was defined in Eq. 21, is the proper variable that combines these two parameters. The filtration efficiency plotted vs. the Stokes number, St, and the modified number, St', are both shown in Figure 16. The experimental results as a function of St' are compared to the theoretically predicted curve using trajectory computations for the total bed composed of nine layers of spheres. Also shown in the figure is the curve predicted by trajectory computations for the single sphere efficiency, which has been converted to total bed efficiency using Eq. 9a. As seen, the experimental results agree well with those predicted by both models for the range of Stokes numbers between 0.01 > St > 0.02. For St' > 0.02 the model seems to overestimate the efficiency. The difference may possibly be explained by the fact that at higher Stokes numbers the Reynolds numbers are also higher due to larger

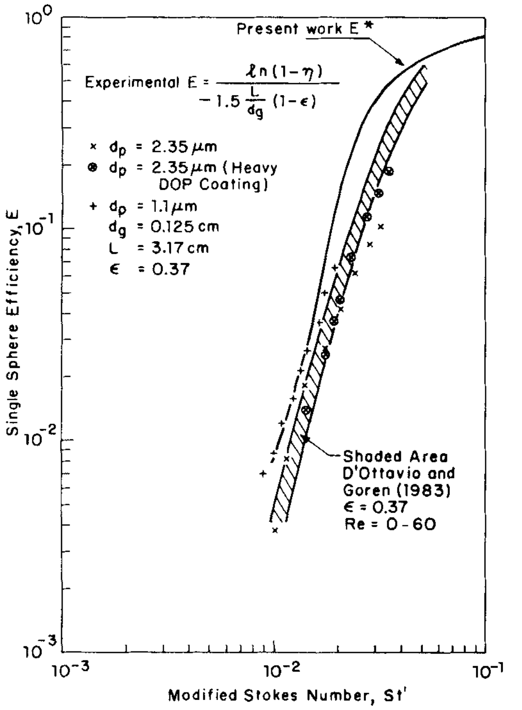

Figure 20. Single sphere efficiency as predicted for a dense cubic packing compared to an empirical correlation for a randomly packed bed $\epsilon = 0.37$, $Re = 0$–60; from D'Ottavio and Goren (1983) and experimental data.

gas velocities and the computed flow field becomes less relevant. Clearly, the helium experiments, which were performed at lower Reynolds numbers, agree better with the model (the upper curve in Figure 16) than the experiments performed with air. Bouncing may also be a factor in the lower experimental efficiencies observed. The coating of the surfaces of the spheres by spraying DOP before the experiment reduces bouncing significantly (see Figure 14) but may not eliminate bouncing completely. Bouncing increases as the kinetic energy of the solid dust particle increases (D'Ottavio and Goren, 1983). The bouncing is also more apparent when the filtration efficiency is high. Thus, at higher values of the parameter St', where the kinetic energy of the dust particle is high, the deviation of the experimental results from the model predictions should be more significant.

A comparison between the experimental results and the single sphere efficiency, E^*, as predicted by trajectory computations is given in Figures 17 and 18. Experimental results using 3-, 9-, and 15-layer beds are shown. The single sphere efficiency for constant St' is independent of the number of layers in the bed. In Figure 17 Eq. 9a or 8 with $K_1 = 1.5$ is used since they both give approximately the same result. The agreement with the predicted curve up to $St = 0.02$ is excellent. For $St \gg 0.02$ the experimental results are lower than predicted as discussed above. In Figure 18, Eq. 8 with $K_1 = 1.5/\epsilon$ is used. The large difference between the predicted and experimental results suggests a significant inaccuracy in using $K_1 = 1.5/\epsilon$ in Eq. 8.

Lastly, the theoretical model is compared to filtration efficiencies obtained in randomly packed beds with different granule sizes and porosities. In a recent paper, D'Ottavio and Goren (1983) found an experimental correlation between the effective Stokes number St_{eff} defined as

$$St_{\text{eff}} = f(Re,\epsilon) \times St/2 \tag{27}$$

and the filtration efficiency. Their experiments were performed using randomly packed beds with $Re = 13$–1620 and porosities $\epsilon = 0.33$–0.38. A comparison between D'Ottavio and Goren's correlation (given in Table 1) for $\epsilon = 0.33$ (the lowest porosity in their experiments) and our theoretical predictions for the single sphere efficiency in a dense cubic packing is shown in Figure 19. As seen, there is a good fit between the two correlations for Reynolds numbers as high as $Re = 60$; as the Reynolds number increases to 140, however, there is an appreciable discrepancy between the two.

Results for experiments using randomly packed beds of glass spheres ($\epsilon = 0.37$, $d_g = 0.125$ cm) to filter 1.1 and 2.35 μm latex particles are shown in Figure 20. There is a reasonably good agreement between the experimental data and the theory for both particle sizes.

NOTATION

a	= filter element radius
A_0	= cross-sectional area of a unit cell
A_c	= area in the base of a unit cell from which particles are collected
A_p	= open area in the base of a unit cell from which air flows into the cell
b	= distance of limit trajectory from X_1 axis
c	= volumetric concentration of aerosol in air
c_o	= initial dust concentration
C	= Cunningham correction factor
C_{ij}, C_{pi}	= constants in Eqs. 10 and 11
d_c	= dimensionless constriction diameter of constricted tube
d_g	= filter element diameter
d_p	= dust particle diameter
e	= radius of unit cell defined in Eq. 4
E	= single sphere efficiency
E'	= single unit cell efficiency
E^*	= single sphere efficiency defined in Eq. 25
f	= collision factor
F	= dimensionless parameter defined in Eq. 20
$\vec{F}_{\text{ext}}$	= external force acting on particle
k	= factor in Eq. 19
K_1	= constant in Eq. 6
ℓ	= characteristic length of unit cell defined by Eq. 9b
L	= bed thickness
m	= number of unit cells
m_p	= mass of dust particle
n	= number of sphere layers
n'	= number of equivalent unit cells defined in Eq. 9c
n_{out}	= concentration of dust particles in filter outlet
n_{in}	= concentration of dust particles in filter inlet
P	= dimensionless pressure
$\Delta p = \Delta P/m$	= pressure drop across a unit cell in granular bed
ΔP	= pressure drop across a granular bed
r	= distance from sphere's center
r_p	= aerosol (dust particle) radius
$R = r/a$	= dimensionless distance from sphere's center
Re	= Reynolds number, $Re = Ud_g\rho_g/\mu$
$R_p = r_p/a$	= interception parameter
St	= Stokes number
St'	= modified Stokes number defined in Eq. 21
t	= time
$T = tUF/a$	= dimensionless time
$\vec{u}$	= air velocity inside a granular bed
$\vec{u}_p$	= dust particle velocity
U	= superficial gas velocity in the z direction
$\vec{U} = \vec{u}/UF$	= dimensionless gas velocity

U_r	= radial velocity
U_{po}	= initial dust particle velocity outside the bed
v_j (j = 1,2,3)	= dimensionless air velocity defined in Eq. 10
V	= dimensionless superficial velocity defined in Eq. 19
V_o = 0.000258	= dimensionless average velocity in the z direction for $Re = 0$. Experimental value from Martin et al. (1951)
$V^*(Z)$	= dimensionless superficial velocity in the Z direction defined in Eq. 15
w_i, w_i'	= weight functions defined in Eqs. 13 and 14
$\vec{x}$	= position vector
x,y,z	= coordinates
x_j (j = 1,2,3)	= dimensionless coordinates
$\vec{X} = \vec{x}/a$	= dimensionless position vector
$X = x/a$, $Y = y/a$, $Z = z/a$	= dimensionless coordinates

Greek Letters

β	= flow intensification factor
ϵ	= void fraction of a granular bed
η	= total filtration efficiency of a granular bed
μ	= gas viscosity
ρ_g	= gas density
ρ_p	= particle density
ϕ_{ji}, ϕ_{pi}	= trigonometric functions that satisfy the periodicity and symmetry of the flow in Eqs. 10 and 11
χ	= a function that satisfies the no-slip condition of flow in Eq. 10
τ	= volume of a unit cell

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